Quantum Information Processing Devices (QIPD) PHYS 770 / ECE 771

Exercises related to teh NMR lectures

Answer 5 of the 6 following questions.

Exercise 1: Show that a generic one qubit rotation can be obtain by combining a generic rotations around Z with $\theta = \pi/2$ rotations around X, i.e. a rotation of the form $e^{-i\pi/4X}$.

Exercise 2: a) Calculate the expectation of the magnetic moment $\vec{\mu}$ in a constant magnetic field in the z direction as a function of time, i.e. $\mathrm{Tr}[\vec{\mu} U \rho_i U^\dagger]$, for $U = exp(-i\hbar\vec{\mu}\cdot\vec{B}t)$ and $\rho_i = (\mathbb{1}+\mathcal{O})/2$ for $\mathcal{O} = X, Y, Z$ and draw on the Block sphere. b) Find the Larmor frequency of a proton in a 16.4 T magnet.

Exercise 3: Show that if $\frac{\pi}{2} \rightarrow \frac{\pi}{2} + \epsilon$, in the robust 3 rotation gate is robust against miscalibration (variation of the angle of rotation) if the inital state is of the form $\rho = 1/2(\mathbb{1} + Z)$ but not if Z is replaced by X pr Y. Sow that the robust 5 rotation gate remains precise to order ϵ^2 for all initla conditions.

Exercise 4: Plot the amount of rf homogeneity selected by the rf selection sequence, for a distribution of $\phi + i$ being a dicretization of a gaussian distribution which includes 4,8,16,32 and 64 points.

Exercise 5: Fourier transform the time evolution of the magnetization of two spins coupled through J-coupling with Hamiltonian of the form $H = \omega_1 Z_1 + \omega_2 Z_2 + \frac{\pi}{2} J Z_1 Z_2$ for initial states of the form $\rho_i = \frac{1}{4}(\mathbb{1} + \mathcal{O}$ for $\mathcal{O} = X^1, Y^1, Z^1$ and $X^1 Z^2$ and give a diagram of the result. Repeat when the $T2$ effect is included, that is multiply the evolution of the magnetization by e^{-t/T_2}.

Exercise 6. Show that the control-not implemented using the NMR gates is indeed a control-not by calculating the effect of the gates (note that there the NMR implementation includes an irrelevant global phase). Do this by straightforwardly multiply the matrices corresponding to each gates. Do the same problem using operator notation, i.e. where for example a rotation of $\pi/2$ around teh X axis of the first qubit is given by $(\mathbb{1} - iX) \otimes \mathbb{1}/\sqrt{2}$.