IQC Institute for Quantum Computing

# Quantum Information Processing Devices, E&CE 770, PHY 771

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**Abstract**

This course introduces the fundamental concepts and the most recent achievements in the physical realization of quantum information devices and systems in three platforms; Nuclear Magnetic Resonance (NMR), quantum photonics and superconducting electrical circuits.

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# Plan

- **1. Introduction R.L. 1.5**
  - Classical & quantum information
  - Quantum information principles (qubit processing and measurement)
  - Quantum circuits
  - Advantages of quantum information
  - DiVincenzo criteria for quantum processors
  - QIP devices
- **2. Nuclear Magnetic Resonance (NMR) QIP R.L. 4.5**
  - Basics of NMR
  - Principles of liquid state NMR QIP
  - Examples of quantum algorithms for NMR
  - Quantum error correction
  - Solid state NMR QIP

- **3. Photonic QIPD G.W. 6**
  - Overview of QED (Quantum Electrodynamics)
  - Encoding quantum information into optical fields
  - Linear optics quantum computing
  - Non linear optics quantum computing
  - Photonic sources of entanglement
  - Quantum optical measurement schemes
  - Prospects of integrated optics in optical quantum computing
- **4 Photonic Quantum Detection H.M. 6**
  - Quantum description of light
  - Classical, semi-classical and quantum description of homodyne and heterodyne detection
  - Noise mechanisms in optical detection
  - Transition from optical detectors to single photon optical detector
  - Semiconductor and superconductive single photon optical detectors
  - Characterization of single-photon optical detectors

- **5 Superconducting Quantum Circuits F.W. 6**
  - Macroscopic degree of freedom in superconducting mesoscopic circuits
  - Foundation and operation of flux qubit structures
  - Foundation and operation of charge number qubit structures
  - Superconducting cavity QED electrical circuits
  - Decoherence in superconducting quantum circuits
- **6 Photonic Quantum Cryptography N. L. 6**
  - Encoding quantum information in propagating optical fields
  - Quantum communication protocols
  - Photonic quantum key distribution
  - Free-space and fiber optical quantum cryptography
  - Most recent experimental achievement and available commercial systems for photonic QKD and quantum communications

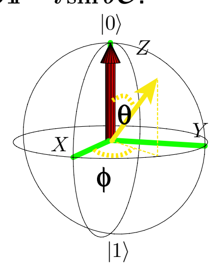
# Notation

We are assuming that you have done or know basic quantum mechanics (postulates, spin 1/2 particles, Pauli matrices, unitary operators).
•Classical information: 01011
•Quantum information: $\Psi = \alpha|0\rangle + \beta|1\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$
•Pauli matrices:

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

•Tensor product $X_k = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes X \otimes \cdots \otimes \mathbb{1}$.
For operators $\mathcal{O}$ such that $\mathcal{O}^2 = \mathbb{1}$, we have $e^{-i\theta\mathcal{O}} = \cos\theta\mathbb{1} - i\sin\theta\mathcal{O}$.

•Bloch sphere: a geometric representation of the state
$\Psi = \cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$ or
$\rho = \frac{1}{2}(\mathbb{1} + \sin[\theta]\cos[\phi]X + \sin[\theta]\sin[\phi]Y + cos[\theta]Z)$




•$\hbar = 1.05 \times 10^{-34}$ Js,
•$k = 1.38 \times 10^{-23}$ J/K

# Main reference

“Introduction to Quantum Information Processing”,
E. Knill, R. Laflamme,
H. Branum, D. Dalvit, J. Dziarmaga,
J. Gubernatis, L. Gurvits,
G. Ortiz, L. Viola
Los Alamos Science
Number 27 p. 2-37, or
quant-ph/0207171, [2]


This can also be found at
www.iqc.ca/publications/tutorials/iqip.pdf

# Classical and quantum information

Classical informaton is encoded in bits: they are units of information that are represented by 0 and 1. Most electronic information processing today can be thought of as either sending a string of bits $10100100001\ldots$ (for communication) or transforming a string of bits into another one $10100100001\cdots \longrightarrow 01111000011\ldots$.
This information can be manipulated through a set of transformations or gates. Physical devices can rarely make the desire transformation at once, but builds it from a set of basic blocks from which we can make any desired transformation. In that case we call this set a universal set.

$$0 \longrightarrow \text{not}(0) = 1$$
$$1 \longrightarrow \text{not}(1) = 0$$

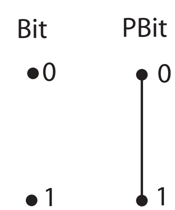
a reset gate

$$0 \longrightarrow \text{reset}(0) = 0$$
$$1 \longrightarrow \text{reset}(1) = 0$$

and a nand gate ($AB \longrightarrow (\bar{AB})B$)

$$00 \longrightarrow \text{nand}(00) = 10$$
$$01 \longrightarrow \text{nand}(01) = 11$$
$$10 \longrightarrow \text{nand}(10) = 10$$
$$11 \longrightarrow \text{nand}(11) = 01$$

from these gates we can build any transformation that you would like.
The bit we have defined is deterministic, the classical system is either in the state 0 or in 1. There is a more general bit, the probabilistic bit, where we assign a probability $p$ to be in the state 0 and a probability $(1-p)$ to be in 1.



# Quantum Information

The foundations of an information-processing theory can be constructed by the following procedure:

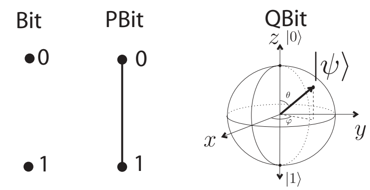
- 1. Define the basic unit of information.
- 2. Give the means for processing one unit.
- 3. Describe how multiple units can be combined.
- 4. Give the means for processing multiple units.
- 5. Show how to convert the content of any of the extant units to classical information.

- Qubits

  The unit of quantum information is the quantum bit (or qubit). It is the information that can be encoded in a system describe by a set of operators represented by the unit matrix $\mathbb{1}$ supplemented by the Pauli matrices $X, Y, Z$. Quantum two level systems such as spin half particles, or physical systems restricted to two energies are good examples.

  Their states is described by

$$\rho = \frac{1}{2}\mathbb{1} + \alpha X + \beta Y + \gamma Z$$




- One qubit gates
  - A generic unitary one qubit gate is describe by a rotation of the Block sphere:

$$U_1 = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}}$$

  The operator $X$ is often called a bit flip.

  The operator $Z$ is often called a phase flip.

  Exercise: Show that a generic one qubit rotation can be obtain by combining a generic rotations around $Z$ with $\theta = \pi/2$ rotations around $X$.

  - A generic one bit gate is not-unitary and is represented by

$$\rho \rightarrow \sum_i A_i \rho A_i^\dagger$$

  where $A_i$ are two by two complex matrices restricted by

$$\sum_i A_i^\dagger A_i = \mathbb{1}$$

- Combining qubits

  The Hilbert space (the space of possible states) of a multiqubit system is described by a tensor product of the states of qubits.

  Some states however can not be written as

$$|\Psi\rangle_{1,2} = |\Phi\rangle_1 \otimes |\chi\rangle_2$$

  but rather as

$$|\Psi\rangle_{1,2} = \sum_{ij} \alpha_{ij} |\Phi_i\rangle_1 \otimes |\chi_j\rangle_2$$

  These have interesting properties, e.g. states that cannot be written in the form $|\Psi\rangle_{1,2} = |\Phi\rangle_1 \otimes |\chi\rangle_2$ have correlations that violates our classical intuition.

- **Multiqubit gates**
  Theorem: **A generic unitary transformation can be decomposed using a univeral set of gates.**

  **The set which consist of a generic one qubit gate**

$$U = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}}$$

  **and Control-Not ($CNot$) is universal**

$$CNot = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

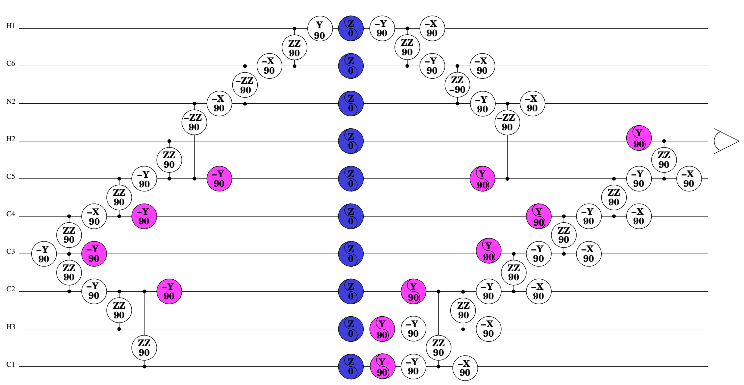
- **Measurement**
  **Measurements of quantum states transform quantum information into classical one. It is described by a projection operator into a basis of state of the multiqubit system.**
  **The result of a measurement is to project the state $|\psi_i\rangle$ in one basis state and this occur with probability**

$$Prob(|\psi_i\rangle) = \mathrm{Trace}[\rho|\psi_i\rangle\langle\psi_i|]$$

  **For one qubit with state $\alpha|0\rangle + \beta|1\rangle$, measuring in the $Z$ basis, will give the state $|0\rangle$ with probability $\alpha\alpha^*$ and the state $|1\rangle$ with probability $\beta\beta^*$.**
  **Measurements turn quantum information into classical information.**

# Quantum circuits

**The evolution of quantum systems can be described by a set of gates. A quantum circuit is a pictorial description of this evolution.**



# Advantage of quantum information

**The fundamental idea of quantum information processing is to use quatum mechanics to manipulat information. Depending on the task at hand, the advantages are for:**

- **Computation: task can be implemented that require less amount of resources (than what is known using classical methods).**
- **Communication: for certain tasks QM gives a provably exponential gain on the amount of ressources required**
- **Cryptography: can insure that eavesdropping has not occurred**
- **Measuring apparatus: increase the sensitivity without requiring additional resources.**

**The required resources is the key to understand the advantage of quantum information.**

# Requirements for Q. Computers [1]

DiVincenzo suggested a set of sufficient element to build a quantum computer

- Hilbert space and subsytems
- Initial state
- Control
- Measurement
- Low noise and decoherence

They are guiding elements for experimentalists.

# Accuracy threshold theorem

A quantum computation can be as long as required with any desired accuracy $e$ as long as the noise level is below a threshold value:

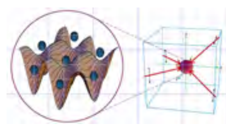
$$P_{\mathrm{error}} < P_{\mathrm{threshold}}$$

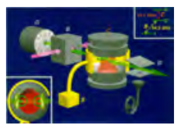
while using only a polynomial amount of resources as a function the size of the original problem and $\log 1/e$

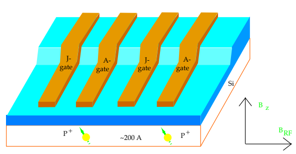
This theorem tell us that is possible to control quantum system in a scalable way and guides us on how low the decoherence has to be.

# Devices for QIP

- Atom traps
- Cavity QED
- Electron floating on helium
- Electron trapped by surface acoustic waves
- Ion trap
- Nuclear Magnetic Resonance
- Quantum dots
- Quantum optics
- Solid state
- Spintronics
- Superconducting Josephson junctions







## References


[1] D.P. DiVincenzo. The physical implementation of quantum computation. Fort. Phys., 48:771–783, 2000.

[2] E. Knill, R. Laflamme, H. Barnum, D. Dalvit, J. Dziarmaga, J. Gubernatis, L. Gurvits, G. Ortiz, L. Viola, and W. Zurek. Introduction to quantum information processing. Los Alamos Science, 27:2–37, 2001. quant-ph/0207171.