Control: one bit gate

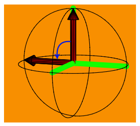
$$H = \sum_i \vec{\mu}_i \cdot \vec{B} = \sum_i \omega_L^i Z_i$$

- The background field $\vec{B}_b = B_b \hat{z}$: B_b around 10 Tesla
- To induce one bit gates use the coil to send rf waves

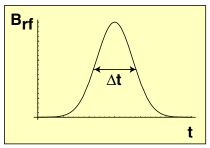
$\vec{B}_{rf} = B_x \cos[\omega_L(t-t_0)]\hat{x} + B_y \sin[\omega_L(t-t_0)]\hat{y}$ which looks like a constant magnetic field in the rotating frame.

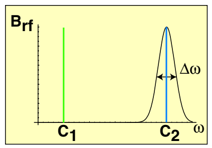
Rotation around x/y axis: e.g. around x
Rotation around z axis:

hard pusle: 10 µs;
soft pulse 1/δ



$$e^{-i\theta X} = \mathbb{1}\cos[\theta] - iX\sin[\theta]$$
$$Z \to Z\cos[2\theta] - Y\sin[2\theta]$$
$$Y \to Y\cos[2\theta] + Z\sin[2\theta]$$





A universal set of one bit gates

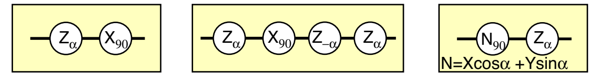
In NMR, the background Hamiltonian provides rotation around the z axis, and only rotation of $90°$ around the x-axis are necessary to obtain universality.

A generic rotation can be written as: $e^{-i\theta\vec{n}\cdot\vec{\sigma}}$ which can be rewritten as

$$e^{-i\frac{\alpha}{2}Z}e^{i\frac{\pi}{4}X}e^{i\frac{\beta}{2}Z}e^{-i\frac{\pi}{4}X}e^{-i\frac{\theta}{2}Z}e^{i\frac{\pi}{4}X}e^{-i\frac{\beta}{2}Z}e^{-i\frac{\pi}{4}X}e^{-i\frac{\alpha}{2}Z}$$

$$\alpha = \tan^{-1}[n_y/n_x] \ : \ \beta = \tan^{-1}[n_x/n_z]$$

In NMR, we have rotation around axis on the $X-Y$ plane and Z rotations come for free as they can be done by redefining subsequent rotation axis of rotation:



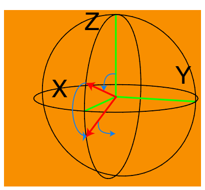
we usually limit ouserlves to $90°$ or $180°$ as they are easier to calibrate.

Pulse with increased robustness

Suppose we are not able to calibrate very well and a $90°$ is really $90° + \epsilon$, how can we make a pulse which reduce this imprecision?

For Z initial state, use the sequence:

$$U = e^{-i(\frac{\pi}{4}+\epsilon)Y}e^{-i(\frac{\pi}{2}+2\epsilon)X}e^{-i(\frac{\pi}{4}+\epsilon)Y}$$



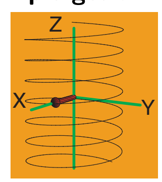
For a general state we need the sequence

$$U = e^{-i\frac{\pi}{2}X}e^{-i\frac{\pi}{2}(X\cos[-30]+Y\sin[-30])}e^{-i\frac{\pi}{2}(X\cos[60]+Y\sin[60])}$$
$$e^{-i\frac{\pi}{2}(X\cos[-30]+Y\sin[-30])}e^{-i\frac{\pi}{2}X}$$

Exercise: Show that if $\frac{\pi}{2} \to \frac{\pi}{2}+\epsilon$, the gate remains precise to order ϵ^2.

Control: gradient fields

Remember $H_{grad} = \vec{\mu}\cdot\vec{B}_{grad}$ and use a gradient field $\vec{B}_{grad} = B_{grad} z\hat{z}$
The sample get a linear phase as a function z:



$$I_\pm(z) = X \pm iY \to e^{-(\pm)i\mu B_{grad} z t} I_\pm$$

The operators $I_\pm$ gets averaged over z.

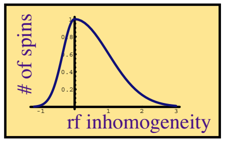
- We can use this to "label" parts of the density matrix, e.g. the one with different number of $I_\pm$.

$$I_+^1\mathbb{1} \to e^{-i\mu B_{grad}z}I_+ \ ; \ I_+^1 I_+^2 \to e^{-i2\mu B_{grad}z}I_+^1 I_+^2$$
$$I_+^1 I_-^2 \to I_+^1 I_-^2 \ ; \ Z^1\mathbb{1} \to Z^1\mathbb{1}$$

$$< I_\pm >_z = \int_{-z_l/2}^{z_l/2} dz e^{-i\mu B_{grad}z} I_\pm \sim \frac{1}{z_l}$$

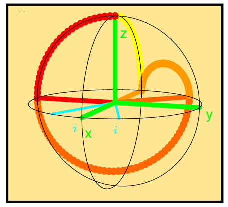
- Gradients can also be used to implement decoherence.

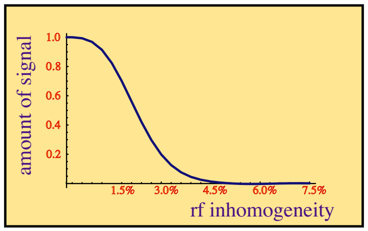
Control: rf selection



The rf power is different for various parts of the sample: we need to find a way to either homogenize or use a sub-sample of the spin where this inhomogeneity is reduced

$$R_x^{90}(R_{-X}^{180})^{64}(R_{\phi_i}^{180}R_{-\phi_i}^{180})^{64}R_Y^{90} \text{ + Gradient}$$
$$\sum_i \phi_i = \pi/8$$





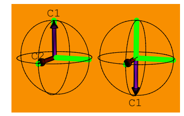
Get homogeneity up to +/- 2% (with around 12% of the signal)

Control: two bit gates

- **Indirect interaction between spins (mediated through electrons)**

$$H = \sum_{ij} J_{ij}\vec{\sigma}^i \cdot \vec{\sigma}^j = \sum_{ij} J_{ij}(X^iX^j + Y^iY^j + Z^iZ^j)$$

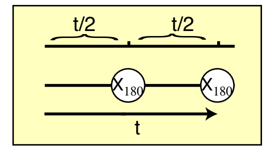
If $|\omega_L^i - \omega_L^j| \ll J_{ij}$, we can neglect $X^iX^j + Y^iY^j$



$$e^{-i\phi ZZ} = 11\!\!1 \cos\phi - iZZ\sin\phi$$
$$X1\!\!1 \rightarrow X1\!\!1 \cos 2\phi + YZ\sin 2\phi$$
$$Y1\!\!1 \rightarrow Y1\!\!1 \cos 2\phi - XZ\sin 2\phi$$

- **Eliminate the natural evolution using refocusing**

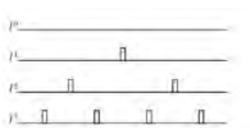
$$1\!\!1 = \underbrace{e^{i\pi X/2}e^{-i\pi J_{12}tZ^1Z^2/2}e^{-i\pi X/2}}_{e^{i\pi J_{12}tZ^1Z^2/2}}e^{-i\pi J_{12}tZ^1Z^2/2}$$



Can we refocus efficiently?

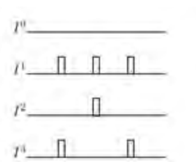
Control: efficient refocusing

If all the qubits are coupled, expectation is that it is not possible, but if we have only two qubit coupling:



This is not efficient, instead use Hadamard matrices, (put a refocusing at each change of sign in a row)

$$\begin{pmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{pmatrix}$$



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