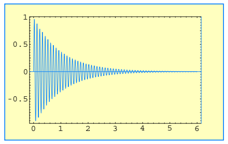
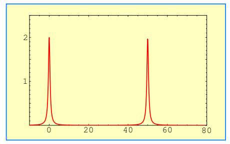
Measurement

For two spins, the magnetization $M(t) = \mathrm{Tr}[\rho(t)(\sigma_-^1 + \sigma_-^2)]$

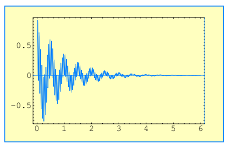
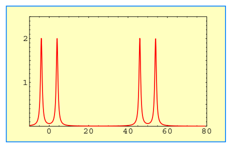
$$\rho(t) = e^{-it(\omega^1 Z_1 + \omega^2 Z_2)t}\rho(t_0)e^{it(\omega^1 Z_1+\omega^2 Z_2)t}$$

The Fourier transform has a peak at the Larmor frequency. The integral of the peak gives us the scale of the matrix element, the width estimates T_2.

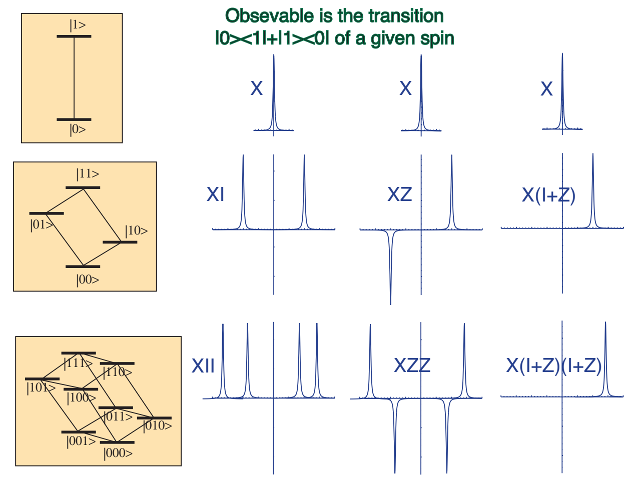
•In the presence of coupling,

$$\rho(t) = e^{-it(\omega^1 Z_1+\omega^2 Z_2+JZ_1Z_2)}\rho(t_0)e^{it(\omega^1 Z_1+\omega^2 Z_2+JZ_1Z_2)}$$

Coupling between qubits is seen has the splitting of the lines in two corresponding to having the second qubit is in the state $|0\rangle$ or $|1\rangle$

Interpretation of spectra



Initial state (1)

The initial state is $\rho = e^{-\beta H}/\mathrm{Tr}e^{-\beta H}$ with $\beta\omega_L \sim 10^{-5}$ implies

$$\rho \approx \frac{1}{N}(\mathbb{1} - \beta\sum_i \omega_i Z_i)$$

Schulman and Vazirani [4]: concentrate polarization of the qubits. A 3 qubit example: if $H = \omega(Z_1 + Z_2 + Z_3)$, we can increase polarization by swapping the states $|011\rangle \longleftrightarrow |100\rangle$

$$\rho^d_{\mathrm{thermal}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-3 \end{pmatrix} \Longleftrightarrow \rho^d_{\mathrm{pol}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-3 \end{pmatrix}$$

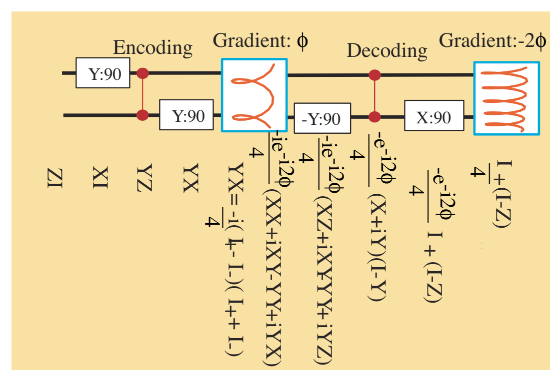
$$\bar{\rho}^d_{\mathrm{pol}} = \mathrm{Tr}\rho^d_{\mathrm{pol}} \approx \frac{3}{4}\beta\omega\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}$$

The initial state (2)

$$\rho = \frac{1}{Z}e^{-\beta H} \sim \frac{1}{Z}(\mathbb{1} - \beta H + \dots)$$

Making a pseudo pure state (Cory et al 1996, Gershenfeld et al. 1997)

$$\frac{1}{Z}(\mathbb{1} - \beta H) \rightarrow \frac{1}{Z}(\mathbb{1} - \frac{\beta\omega n}{2^n}|\Psi\rangle\langle\Psi|)$$



Highly mixed state for QIP

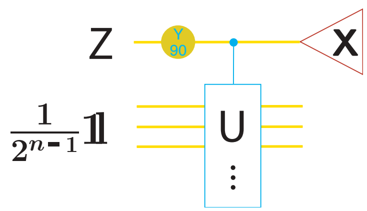
E. Knill& R.L. PRL81, 5672, 1998

$$\rho_i = \frac{1}{Z} e^{-\beta H} \approx \frac{1}{Z}(\mathbb{1} - \beta H + \dots)$$

Making a pseudo pure state on only one bit

$$\rho_i = \frac{1}{N}(\mathbb{1} - \frac{\omega}{kT} Z \otimes \mathbb{1} \otimes \dots)$$

and evolve with $U_{\text{one bit}}$ described the circuit below, measuring X on the first qubit alone will give



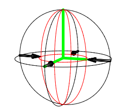
$$\mathrm{Tr}[X_1 U_{\text{one bit}} \rho_i U^\dagger_{\text{one bit}}] = \frac{1}{2^{n-1}} Re(\mathrm{Tr}[U])$$

Noise

The strong magnetic field in the z direction, with some work to homogenize the magnetic in the $X-Y$ direction reduces the number of parameters that describe the noise to three. The temperature, T2 and T1.

T2 describe the rate of decoherence (the randomization of the phase) which induce the decay of the off diagonal terms. It is caused by coupling to other spins or to the inhomogeneity of the magnetic field. It is a unital quantum operation, i.e. preserve the unit matrix.

$$\rho = \begin{pmatrix} a & b+ic \\ b-ic & 1-a \end{pmatrix} \rightarrow \begin{pmatrix} a & (b+ic)e^{-t/T_2} \\ (b-ic)e^{-t/T_2} & 1-a \end{pmatrix}$$



T2 is of the order off seconds in liquid state NMR.

Relaxation to thermal equilibirum is given at a rate called $T1$. It describe the interaction with the "lattice".

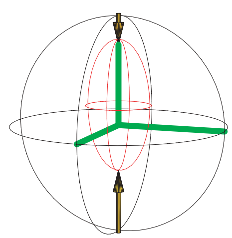
The evolution of a family of isolated spin in pure states evolve as:

$$\rho = \begin{pmatrix} \rho^i_{00} & \rho^i_{01} \\ \rho^i_{10} & \rho^i_{11} \end{pmatrix}$$

$$\rightarrow$$

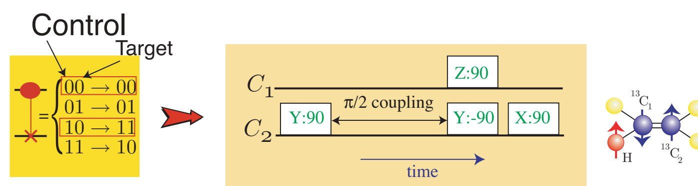
$$\rho = \begin{pmatrix} \rho^{\text{eq}}_{00}(1-e^{t/T_1}) + \rho^i_{00}e^{-t/T_1} & \rho^i_{01}e^{-t/2*T_1} \\ \rho^i_{10}e^{-t/2*T_1} & \rho^{\text{eq}}_{11}(1-e^{t/T_1}) + \rho^i_{11}e^{-t/T_1} \end{pmatrix}$$

with $\rho^{\text{eq}}_{00} = e^{\beta\omega}/(e^{\beta\omega} + e^{-\beta\omega})$ and $\rho^{\text{eq}}_{11} = e^{-\beta\omega}/(e^{\beta\omega} + e^{-\beta\omega})$



T1 for liquid state NMR is of the order of seconds to tens of seconds.

Control-Not from NMR gates



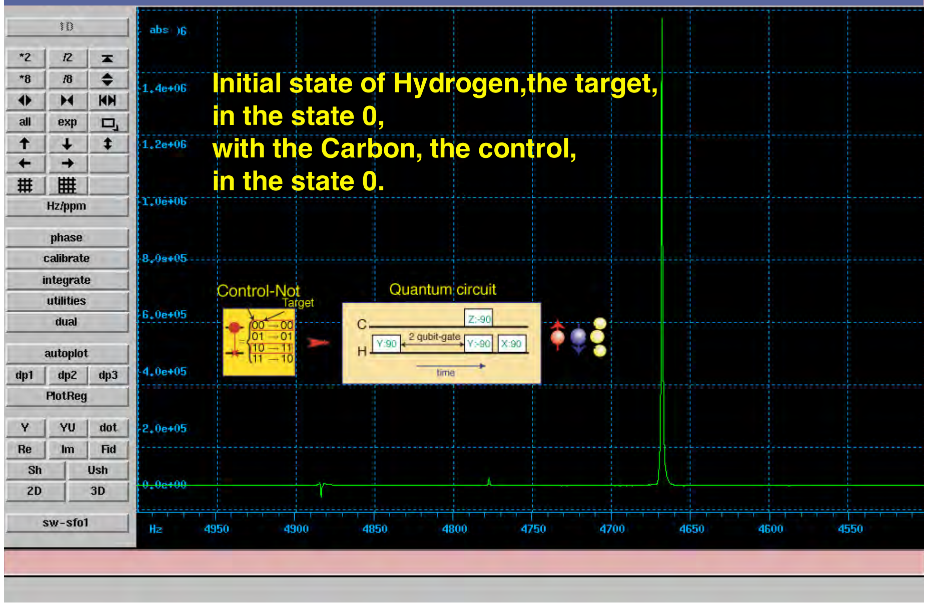
A control-not is implemented by combining one bit gates and a single two qubit gate. C_1 is the control and C_2 the target.

We can translate these gates into the spectrometer language.

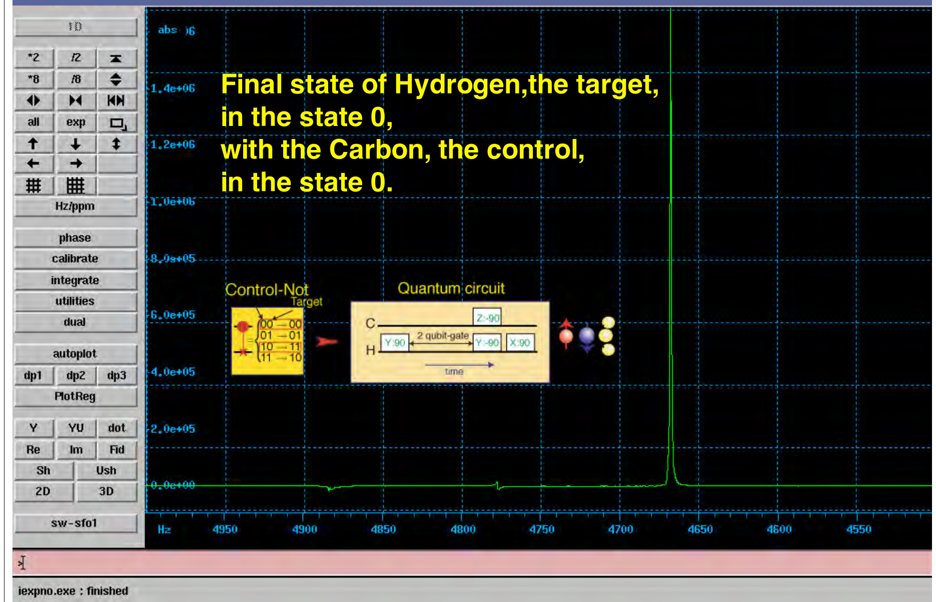
Pulse sequence

```
(C2_90:sp9 ph13 ):f1
 3u
 3u ipp13

 0.71365m

 8u
 8u
(C2_90:sp9 ph19):f1
 6u ipp15 ipp19
 8u
(C2_90:sp9 ph20):f1
 6u ipp15 ipp20
```

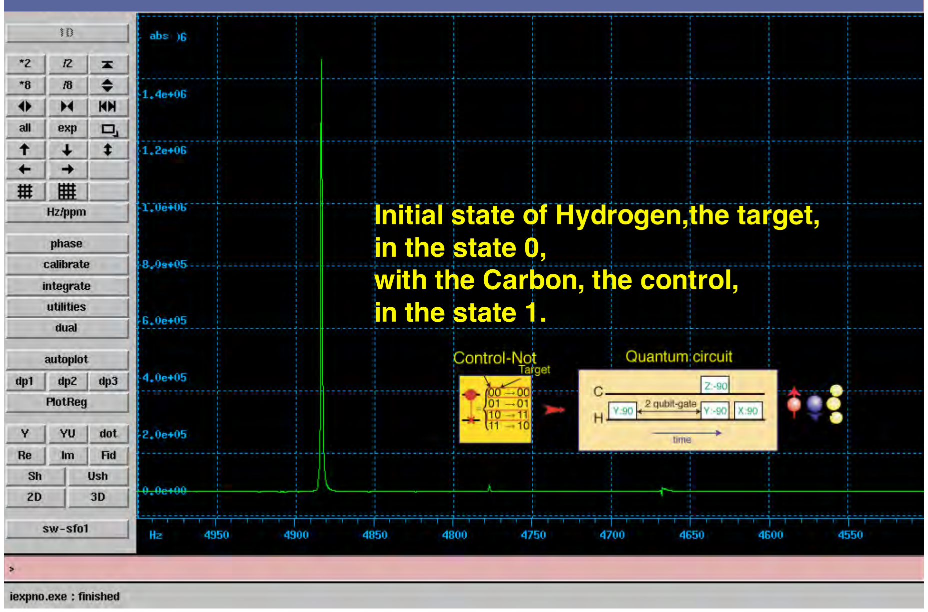
Control-Not at the spectrometer



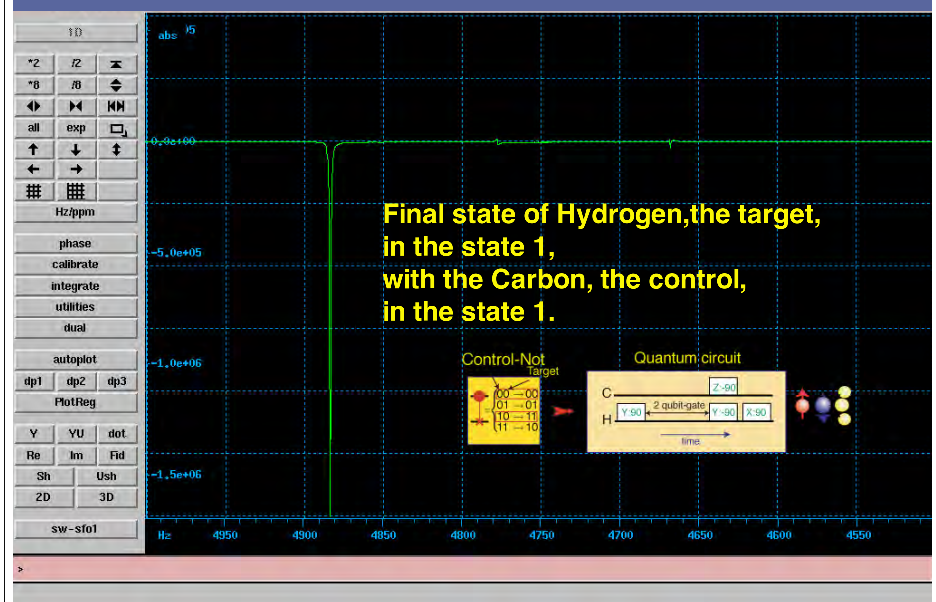
Control-Not at the spectrometer



Control-Not at the spectrometer

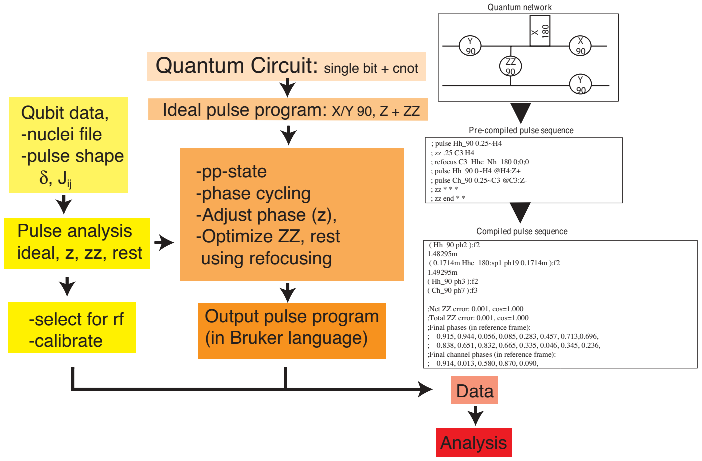


Control-Not at the spectrometer

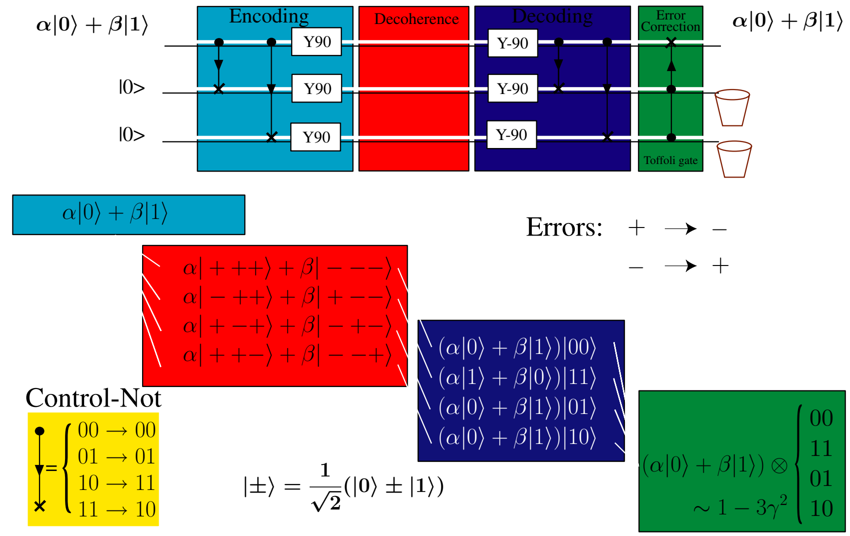


A quantum compiler

as we increase the number of qubits, we need to automate many tasks such as the refocusiign schemes and also deal systematically with errors that can be partially corrected.

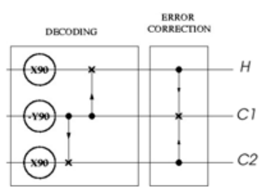


3 qubit code for phase errors

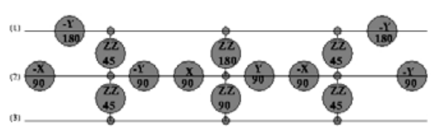


Phase QEC NMR circuit

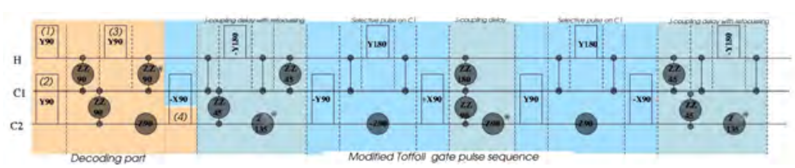
NMR implementation of the decoding and error correction:



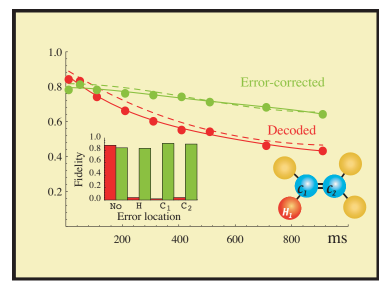
Toffoli gate:



and the full decoding and Toffoli, including some optimization



Experimental results



The demonstration of quantum error correction is in the shape of the green curve which does not have the first order error. The curve is much flatter than the red one.

Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek,T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

Spin half selection

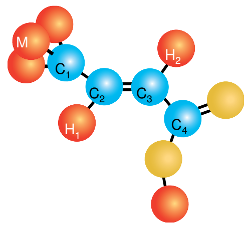
Projection on the spin 1/2

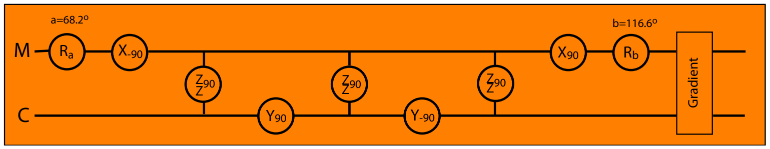
$P_{1/2} = \mathbb{1} - P_{3/2}$

$= \mathbb{1} - [|3/2, 3/2\rangle + |3/2, 1/2\rangle +$
$|3/2, -1/2\rangle + |3/2, -3/2\rangle]$

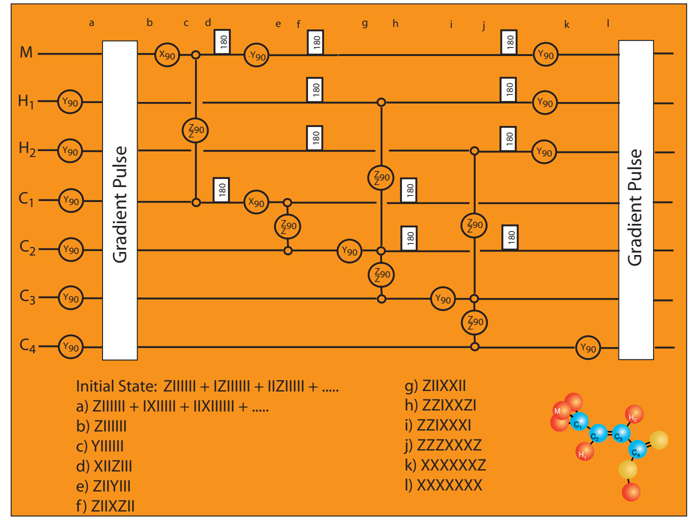
in term of operators we have

$P_{1/2} = [\frac{1}{2}\mathbb{1}\mathbb{1}\mathbb{1} - \frac{1}{6}[(\mathbb{1}XX) + (\mathbb{1}YY) + (\mathbb{1}ZZ)]]$

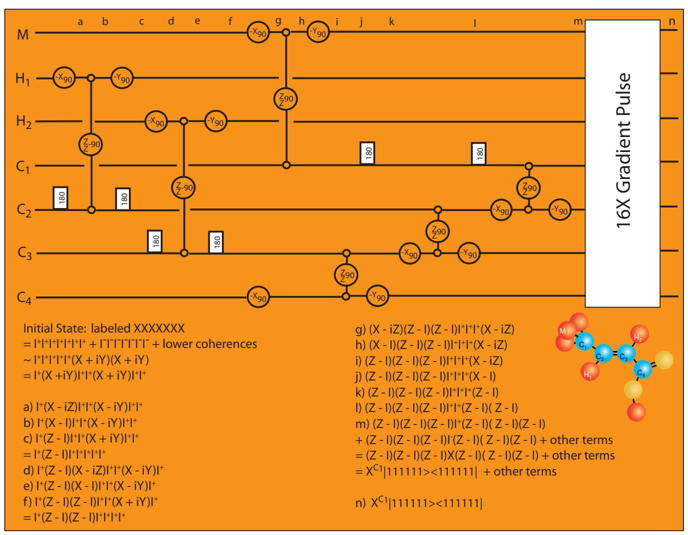




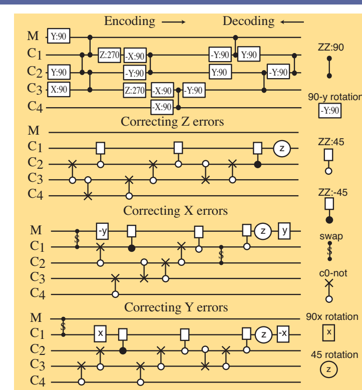
PP state for crotonic acid: encoding



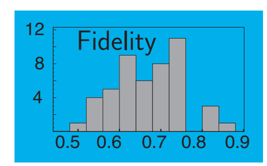
PP state for crotonic acid: decoding

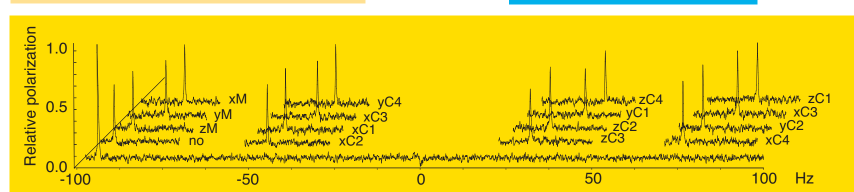


5 bit quantum error correcting code



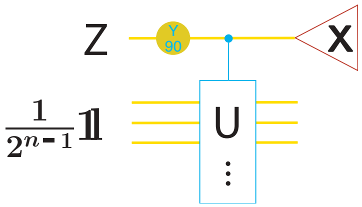
Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors [1].





The power of one qubit

We have seen that the circuit below can be powerful computationnaly



$$\mathrm{Tr}[X_1 U_{\text{one bit}} \rho_i U^\dagger_{\text{one bit}}] = \frac{1}{2^{n-1}} Re(\mathrm{Tr}[U])$$

Are there any interesting U?

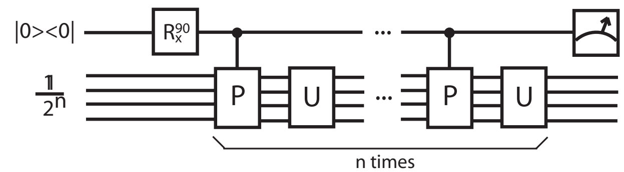
It is possible to characterize complex dynamics. In reference [?] it was shown that the average fidelity decay of the state where we evolve under a unitary evolution UP where U is characteristic of the evolution and P is a perturnation of it

$$F_n(\psi) = \left|\langle\psi| (U^n)^\dagger U_p^n |\psi\rangle\right|^2$$

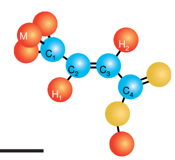
is given by

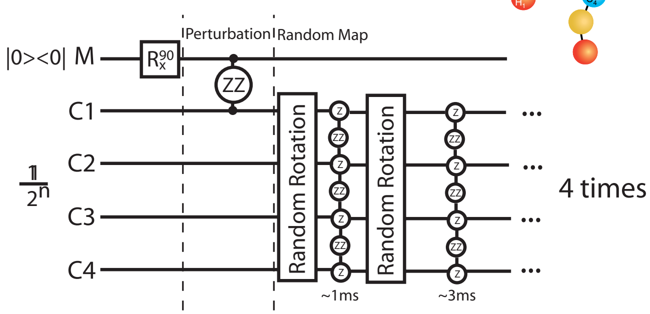
$$\overline{F_n} = \frac{\left|Tr\left[(U^n)^\dagger (PU)^n\right]\right|^2 + N}{N^2 + N}.$$

A simpified circuit to evaluate this is given by (noting that the control on the U can be dropped without changing the circuit):

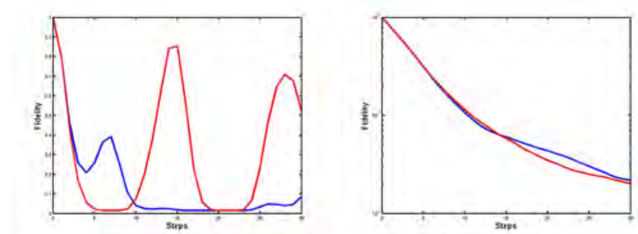


The circuit can be implemented in crotonic acid

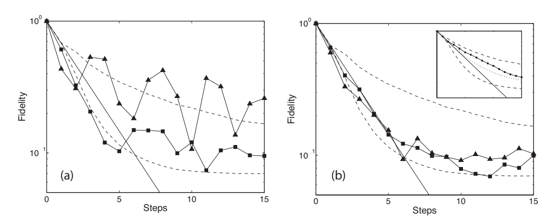




The diagrams below give the behavior of regular and chaotic evolution:

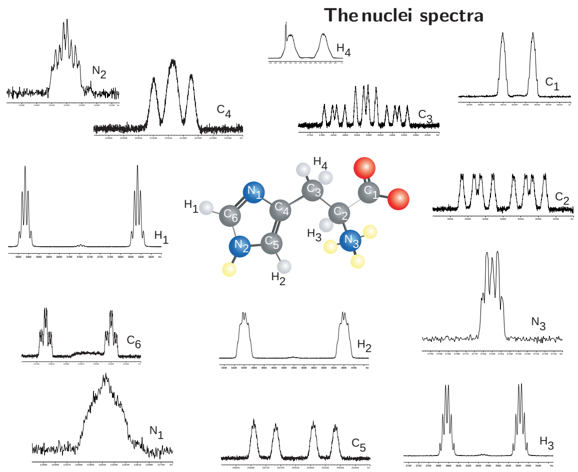


and experimentally we obtain:

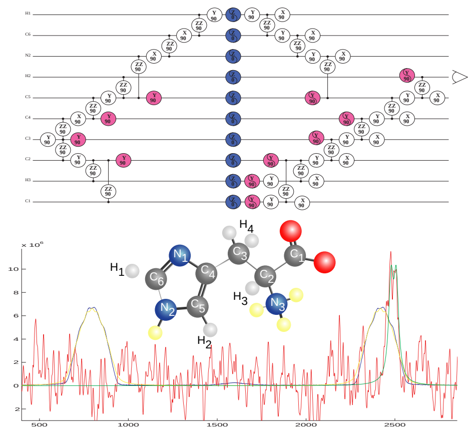


For details see [3]

QIP with histidine



A ten qubits cat-state in histidine



References

[1] E. Knill, R. Laflamme, R. Martinez, and C. Negrevergne. Implementation of the five qubit error correction benchmark. Phys. Rev. Lett., 86:5811–5814, 2001.

[2] R. Laflamme, E. Knill, D. Cory, E. M. Fortunato, T. Havel, C. Miquel, R. Martinez, C. Negrevergne, G. Ortiz, M. A. Pravia, S. Sinha, R. Somma, and L. Viola. Introduction to NMR quantum information processing. Los Alamos Science, (27):226–259, 2001. quant-ph/0207172.

[3] C. Ryan and R. Laflamme. Characterization of complex quantum dynamics with a scalable nmr information processor.

[4] L. J. Schulman and U. Vazirani. Scalable NMR quantum computation. In Proceedings of the 31th Annual ACM Symposium on the Theory of Computation (STOC), pages 322–329, El Paso, Texas, 1998. ACM Press.